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基于 LMI 的不确定随机时滞系统输出反馈保性能控制

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摘要:针对一类范数有界的随机时滞系统,研究在一定性能指标下的保性能控制问题。本文为获得能够实现系统保性能控制的最优输出反馈控制器,采用 Lyapunov 稳定性理论与 LMI 技术相结合的方法,将保性能控制问题转化为 LMIs 约束下的可行解问题。提出的控制器设计方案能够实现系统是依概率全局渐近稳定的,同时满足对系统性能指标的要求。此外,通过数值算例说明所提出控制方案的可行性。

关键词:线性矩阵不等式;保性能控制;随机时滞系统;输出反馈

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随机时滞系统是广泛存在的一类系统,由于受到外部环境或者自身结构的影响,系统往往存在不确定性和时间滞后性,这对系统的稳定性和性能指标产生负面影响。针对随机时滞系统保性能控制的研究有诸多优点,在保证系统稳定性的同时使系统的性能指标不超过某个上界,因此,为满足各类实际工程系统发展的需要,该系统的保性能控制研究引起了广泛关注。保性能控制这一重要思想由文献[1]首次提出,此后有关保性能控制的研究考虑了不确定性、滞后性的影响,并取得了重要研究成果[2—8]。对于随机时滞系统,时滞和随机干扰项的存在增加了研究难度,文献[9—13]主要采用状态反馈的控制方式。但由于实际系统的状态往往难以测得,相比之下,输出反馈能更好地满足实际系统的需要且实施控制成本更低。受文献[14—16]的启发,本文利用随机系统 Lyapunov 稳定性理论和 Itô 公式,针对一类范数有界的随机时滞系统,利用 LMI 方法设计最优输出反馈控制器,使系统在高斯白噪声干扰的影响下实现保性能控制。

考虑如下一类不确定随机时滞系统:

$$\begin{cases}\dot{\boldsymbol{x}}(t)=(\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(t)+(\boldsymbol{A}_1+\Delta\boldsymbol{A}_1)\boldsymbol{x}(t-d)+(\boldsymbol{B}+\Delta\boldsymbol{B})\boldsymbol{u}(t)+(\boldsymbol{G}\boldsymbol{x}(t)+\boldsymbol{u}(t))\boldsymbol{\zeta},\\ \boldsymbol{y}(t)=(\boldsymbol{C}+\Delta\boldsymbol{C})\boldsymbol{x}(t)+(\boldsymbol{C}_1+\Delta\boldsymbol{C}_1)\boldsymbol{x}(t-d)+(\boldsymbol{D}+\Delta\boldsymbol{D})\boldsymbol{u}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),\ \forall t\in[-d,0],\end{cases}\tag{1}$$

其中:$\boldsymbol{x}(t)\in\mathbf{R}^n$,$\boldsymbol{y}(t)\in\mathbf{R}^m$,$\boldsymbol{u}(t)\in\mathbf{R}^l$ 分别是系统的状态向量、输出向量和控制输入,$\boldsymbol{\zeta}$ 是白噪声;$\boldsymbol{A}$,$\boldsymbol{A}_1$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{C}_1$,$\boldsymbol{D}$,$\boldsymbol{G}$ 是已知常数矩阵;$\Delta\boldsymbol{A}$,$\Delta\boldsymbol{A}_1$,$\Delta\boldsymbol{B}$,$\Delta\boldsymbol{C}$,$\Delta\boldsymbol{C}_1$,$\Delta\boldsymbol{D}$ 是未知矩阵,假设它们范数有界,且满足:

$$\begin{bmatrix}\Delta\boldsymbol{A} & \Delta\boldsymbol{B} & \Delta\boldsymbol{A}_1\\ \Delta\boldsymbol{C} & \Delta\boldsymbol{D} & \Delta\boldsymbol{C}_1\end{bmatrix}=\begin{bmatrix}\boldsymbol{H}_1\\ \boldsymbol{H}_2\end{bmatrix}\boldsymbol{F}[\boldsymbol{E}_1\quad \boldsymbol{E}_2\quad \boldsymbol{E}_3],$$

$\boldsymbol{F}$ 是满足 $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\leqslant\boldsymbol{I}$ 的不确定矩阵,$\boldsymbol{H}_1$,$\boldsymbol{H}_2$,$\boldsymbol{E}_1$,$\boldsymbol{E}_2$,$\boldsymbol{E}_3$ 是已知的常数矩阵。

对于系统(1),定义性能指标如下:

$$\boldsymbol{J}=\boldsymbol{E}\int_0^{\infty}[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t)+\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t)]\,\mathrm{d}t,\tag{2}$$

其中,$\boldsymbol{Q}$ 和 $\boldsymbol{R}$ 是对称正定加权矩阵。

文献[14—15]针对随机系统保性能控制的研究提出了有效的设计方法,但没有考虑滞后对于系统

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稳定性的负面影响。考虑到实际工程系统中随机和时滞现象的普遍存在,本文针对不确定性、时滞和高斯白噪声三者对系统的影响,设计能够实现系统保性能控制的最优输出反馈控制器,同时得到系统性能指标的最小上界。

1　预备知识

引理 1[17](矩阵的 Schur 补性质)　对给定的对称矩阵 $\boldsymbol{K}=\begin{bmatrix}\boldsymbol{K}_{11} & \boldsymbol{K}_{12}\\ \boldsymbol{K}_{21} & \boldsymbol{K}_{22}\end{bmatrix}$,以下三个条件是等价的:

(i) $\boldsymbol{K}<0$;

(ii) $\boldsymbol{K}_{11}<0,\boldsymbol{K}_{22}-\boldsymbol{K}_{21}\boldsymbol{K}_{11}^{-1}\boldsymbol{K}_{12}<0$;

(iii) $\boldsymbol{K}_{22}<0,\boldsymbol{K}_{11}-\boldsymbol{K}_{12}\boldsymbol{K}_{22}^{-1}\boldsymbol{K}_{21}<0$。

引理 2[18]　给定矩阵 $\boldsymbol{D}$ 和 $\boldsymbol{E}$,以及对称矩阵 $\boldsymbol{Y}$,则对所有满足 $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\leqslant\boldsymbol{I}$ 的矩阵 $\boldsymbol{F}$,$\boldsymbol{Y}+\boldsymbol{DFE}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}<0$ 成立,当且仅当存在常数 $\varepsilon>0$,使得 $\boldsymbol{Y}+\varepsilon\boldsymbol{DD}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}<0$。

2　输出反馈控制器设计和稳定性分析

2.1　输出反馈控制器设计

设计输出反馈控制器

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)=\boldsymbol{A}_{\mathrm{c}}\hat{\boldsymbol{x}}(t)+\boldsymbol{B}_{\mathrm{c}}\boldsymbol{y}(t),\\ \boldsymbol{u}(t)=\boldsymbol{C}_{\mathrm{c}}\hat{\boldsymbol{x}}(t),\end{cases}\tag{3}$$

其中,$\hat{\boldsymbol{x}}(t)\in\mathbf{R}^{n}$ 是控制器的状态变量,$\boldsymbol{A}_{\mathrm{c}},\boldsymbol{B}_{\mathrm{c}},\boldsymbol{C}_{\mathrm{c}}$ 是待定的具有适当维数的矩阵。

将控制器(3)应用到系统(1),可得

$$\begin{cases}\dot{\boldsymbol{x}}(t)=(\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(t)+(\boldsymbol{BC}_{\mathrm{c}}+\Delta\boldsymbol{BC}_{\mathrm{c}})\hat{\boldsymbol{x}}(t)+(\boldsymbol{A}_{1}+\Delta\boldsymbol{A}_{1})\boldsymbol{x}(t-d)+(\boldsymbol{Gx}(t)+\boldsymbol{u}(t))\boldsymbol{\zeta},\\ \dot{\hat{\boldsymbol{x}}}(t)=(\boldsymbol{B}_{\mathrm{c}}\boldsymbol{C}+\boldsymbol{B}_{\mathrm{c}}\Delta\boldsymbol{C})\boldsymbol{x}(t)+(\boldsymbol{A}_{\mathrm{c}}+\boldsymbol{B}_{\mathrm{c}}\boldsymbol{DC}_{\mathrm{c}}+\boldsymbol{B}_{\mathrm{c}}\Delta\boldsymbol{DC}_{\mathrm{c}})\hat{\boldsymbol{x}}(t)+(\boldsymbol{B}_{\mathrm{c}}\boldsymbol{C}_{1}+\boldsymbol{B}_{\mathrm{c}}\Delta\boldsymbol{C}_{1})\boldsymbol{x}(t-d),\end{cases}$$

所以,闭环系统为:

$$\dot{\bar{\boldsymbol{x}}}(t)=(\bar{\boldsymbol{A}}+\bar{\boldsymbol{H}}\boldsymbol{F}\bar{\boldsymbol{E}})\bar{\boldsymbol{x}}(t)+(\bar{\boldsymbol{A}}_{1}+\bar{\boldsymbol{H}}\boldsymbol{F}\bar{\boldsymbol{E}}_{3})\bar{\boldsymbol{x}}(t-d)+\bar{\boldsymbol{B}}\bar{\boldsymbol{x}}(t)\boldsymbol{\zeta},\tag{4}$$

其中:

$$\bar{\boldsymbol{x}}(t)=\begin{bmatrix}\boldsymbol{x}^{\mathrm{T}}(t) & \hat{\boldsymbol{x}}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},\bar{\boldsymbol{x}}(t-d)=\boldsymbol{x}(t-d),\bar{\boldsymbol{E}}=\begin{bmatrix}\boldsymbol{E}_{1} & \boldsymbol{E}_{2}\boldsymbol{C}_{\mathrm{c}}\end{bmatrix},\bar{\boldsymbol{E}}_{3}=\boldsymbol{E}_{3},$$

$$\bar{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{BC}_{\mathrm{c}}\\ \boldsymbol{B}_{\mathrm{c}}\boldsymbol{C} & \boldsymbol{A}_{\mathrm{c}}+\boldsymbol{B}_{\mathrm{c}}\boldsymbol{DC}_{\mathrm{c}}\end{bmatrix},\bar{\boldsymbol{A}}_{1}=\begin{bmatrix}\boldsymbol{A}_{1}\\ \boldsymbol{B}_{\mathrm{c}}\boldsymbol{C}_{1}\end{bmatrix},\bar{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{G} & \boldsymbol{C}_{\mathrm{c}}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},\bar{\boldsymbol{H}}=\begin{bmatrix}\boldsymbol{H}_{1}\\ \boldsymbol{B}_{\mathrm{c}}\boldsymbol{H}_{2}\end{bmatrix}。$$

据文献[19],系统(4)等价为

$$\mathrm{d}\bar{\boldsymbol{x}}(t)=[(\bar{\boldsymbol{A}}+\bar{\boldsymbol{H}}\boldsymbol{F}\bar{\boldsymbol{E}})\bar{\boldsymbol{x}}(t)+(\bar{\boldsymbol{A}}_{1}+\bar{\boldsymbol{H}}\boldsymbol{F}\bar{\boldsymbol{E}}_{3})\bar{\boldsymbol{x}}(t-d)]\mathrm{d}t+\bar{\boldsymbol{B}}\bar{\boldsymbol{x}}(t)\mathrm{d}\boldsymbol{w}(t),\tag{5}$$

其中,$\boldsymbol{w}(t)$ 是完备概率空间(Ω,F,P)上的标准 n 维维纳过程(布朗运动)。对应闭环系统的性能指标为:

$$J=E\int_{0}^{\infty}\bar{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{Q}}^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{x}}(t)\mathrm{d}t,\tag{6}$$

其中,$\bar{\boldsymbol{Q}}=\begin{bmatrix}\boldsymbol{Q}^{\frac{1}{2}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{R}^{\frac{1}{2}}\boldsymbol{C}_{\mathrm{c}}\end{bmatrix}$。

2.2　系统稳定性分析

定理 1　考虑系统(5)和性能指标(6),若存在对称矩阵 $P,S > 0$,对满足 $F^{\mathrm{T}}F \leqslant I$ 的不确定矩阵 F,以下矩阵不等式成立:

$$\begin{bmatrix} (\bar{A}+\bar{H}F\bar{E})^{\mathrm{T}}P + P(\bar{A}+\bar{H}F\bar{E}) + \bar{B}^{\mathrm{T}}P\bar{B} + \tilde{S} + \bar{Q}^{\mathrm{T}}\bar{Q} & P(\bar{A}_1+\bar{H}F\bar{E}_3) \\ (\bar{A}_1+\bar{H}F\bar{E}_3)^{\mathrm{T}}P & -S \end{bmatrix} < 0, \tag{7}$$

则闭环系统(5)是依概率全局渐近稳定,且系统性能指标(6)满足 $J \leqslant E(\bar{x}_0^{\mathrm{T}}P\bar{x}_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau)Sx(\tau)\mathrm{d}\tau)$。

证明　对闭环系统(5)选取如下形式的 Lyapunov 泛函:

$$V(x(t)) = \bar{x}^{\mathrm{T}}(t)P\bar{x}(t) + \int_{t-d}^{t}\bar{x}^{\mathrm{T}}(\tau)\tilde{S}\bar{x}(\tau)\mathrm{d}\tau,$$

其中,$\tilde{S} = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}$,$P,S$ 是对称正定矩阵。利用 Itô 公式得

$$\begin{aligned} LV(t,x(t)) = {} & \bar{x}^{\mathrm{T}}(t)\tilde{S}\bar{x}(t) - \bar{x}^{\mathrm{T}}(t-d)S\bar{x}(t-d) + \bar{x}^{\mathrm{T}}(t)P(\bar{A}+\bar{H}F\bar{E})\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)(\bar{A}+\bar{H}F\bar{E})^{\mathrm{T}}P\bar{x}(t) + \\ & \bar{x}^{\mathrm{T}}(t)P(\bar{A}_1+\bar{H}F\bar{E}_3)\bar{x}(t-d) + \bar{x}^{\mathrm{T}}(t-d)(\bar{A}_1+\bar{H}F\bar{E}_3)^{\mathrm{T}}P\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)\bar{B}^{\mathrm{T}}P\bar{B}\bar{x}(t) = \\ & \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t-d) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (\bar{A}+\bar{H}F\bar{E})^{\mathrm{T}}P + P(\bar{A}+\bar{H}F\bar{E}) + \bar{B}^{\mathrm{T}}P\bar{B} + \tilde{S} & P(\bar{A}_1+\bar{H}F\bar{E}_3) \\ (\bar{A}_1+\bar{H}F\bar{E}_3)^{\mathrm{T}}P & -S \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t-d) \end{bmatrix}。 \end{aligned}$$

由 $LV(t,x(t)) < 0$ 可得:

$$\begin{bmatrix} (\bar{A}+\bar{H}F\bar{E})^{\mathrm{T}}P + P(\bar{A}+\bar{H}F\bar{E}) + \bar{B}^{\mathrm{T}}P\bar{B} + \tilde{S} & P(\bar{A}_1+\bar{H}F\bar{E}_3) \\ (\bar{A}_1+\bar{H}F\bar{E}_3)^{\mathrm{T}}P & -S \end{bmatrix} < \begin{bmatrix} -\bar{Q}^{\mathrm{T}}\bar{Q} & 0 \\ 0 & 0 \end{bmatrix} < 0,$$

即不等式(7)成立,则随机时滞系统(5)是依概率全局渐近稳定的,且 $LV < -\bar{x}^{\mathrm{T}}(t)\bar{Q}^{\mathrm{T}}\bar{Q}\bar{x}(t)$。对不等式两边同时取期望和积分,得

$$J = E\left(\int_0^{\infty}\bar{x}^{\mathrm{T}}(t)\bar{Q}^{\mathrm{T}}\bar{Q}\bar{x}(t)\mathrm{d}t\right) \leqslant E\left(\bar{x}_0^{\mathrm{T}}P\bar{x}_0 + \int_{-d}^{0}x^{\mathrm{T}}(\tau)Sx(\tau)\mathrm{d}\tau\right),$$

定理得证。

由于不等式(7)不是线性矩阵不等式,因此,利用引理 1 将其转化为等价的线性矩阵不等式可行性问题。

定理 2　针对系统(5)和性能指标(6),给定 $\varepsilon > 0$ 和对称矩阵 $S > 0$,若存在对称矩阵 $X,Y > 0$ 及矩阵 $\hat{A},\hat{B},\hat{C}$,有下面不等式成立

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \tag{8}$$

$$\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & A_1 & X & H_1 & XE_1^{\mathrm{T}} + \hat{C}E_2^{\mathrm{T}} & XG^{\mathrm{T}} + \hat{C}^{\mathrm{T}} & 0 & XQ^{\frac{1}{2}} & \hat{C}^{\mathrm{T}}R^{\frac{1}{2}} \\ * & \Phi_3 & \Phi_4 & I & YH_1 + \hat{B}H_2 & E_1^{\mathrm{T}} & G^{\mathrm{T}} & 0 & Q^{\frac{1}{2}} & 0 \\ * & * & -S & 0 & 0 & \bar{E}_3^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & * & * & -S^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon^{-1}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -X & -M & 0 & 0 \\ * & * & * & * & * & * & * & -Z & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{9}$$

其中,$\boldsymbol{\Phi}_1 = \boldsymbol{AX} + \boldsymbol{XA}^{\mathrm{T}} + \boldsymbol{B}\hat{\boldsymbol{C}} + \hat{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$,$\boldsymbol{\Phi}_2 = \boldsymbol{A} + \hat{\boldsymbol{A}}^{\mathrm{T}}$,$\boldsymbol{\Phi}_3 = \boldsymbol{YA} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Y} + \hat{\boldsymbol{B}}\boldsymbol{C} + \boldsymbol{C}^{\mathrm{T}}\hat{\boldsymbol{B}}^{\mathrm{T}}$,$\boldsymbol{\Phi}_4 = \boldsymbol{YA}_1 + \hat{\boldsymbol{B}}\boldsymbol{C}_1$,则随机时滞系统(5) 的性能指标满足:$J \leqslant E\left(\bar{\boldsymbol{x}}_0^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{x}}_0 + \int_{-d}^{0}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{Sx}(\tau)\mathrm{d}\tau\right)$。

证明　令 $\bar{\boldsymbol{Y}} = \begin{bmatrix} \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}} + \tilde{\boldsymbol{S}} + \bar{\boldsymbol{Q}}^{\mathrm{T}}\bar{\boldsymbol{Q}} & \boldsymbol{P}\bar{\boldsymbol{A}}_1 \\ \bar{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{S} \end{bmatrix}$,则不等式(7)可写为:

$$\bar{\boldsymbol{Y}} + \begin{bmatrix} \boldsymbol{P}\bar{\boldsymbol{H}} \\ \boldsymbol{0} \end{bmatrix}\boldsymbol{F}\begin{bmatrix} \bar{\boldsymbol{E}} & \bar{\boldsymbol{E}}_3 \end{bmatrix} + \begin{bmatrix} \bar{\boldsymbol{E}} & \bar{\boldsymbol{E}}_3 \end{bmatrix}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\begin{bmatrix} \boldsymbol{P}\bar{\boldsymbol{H}} \\ \boldsymbol{0} \end{bmatrix}^{\mathrm{T}} < 0。$$

根据引理 2,得到:

$$\begin{bmatrix} \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}} + \varepsilon\boldsymbol{P}\bar{\boldsymbol{H}}\bar{\boldsymbol{H}}^{\mathrm{T}}\boldsymbol{P} + \varepsilon^{-1}\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}} + \bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}} + \tilde{\boldsymbol{S}} + \bar{\boldsymbol{Q}}^{\mathrm{T}}\bar{\boldsymbol{Q}} & \boldsymbol{P}\bar{\boldsymbol{A}}_1 + \varepsilon^{-1}\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}_3 \\ \bar{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{P} + \varepsilon^{-1}\bar{\boldsymbol{E}}_3^{\mathrm{T}}\bar{\boldsymbol{E}} & -\boldsymbol{S} + \varepsilon^{-1}\bar{\boldsymbol{E}}_3^{\mathrm{T}}\bar{\boldsymbol{E}}_3 \end{bmatrix} < 0。\tag{10}$$

应用引理 1,不等式(10)进一步等价于:

$$\begin{bmatrix} \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}} & \boldsymbol{P}\bar{\boldsymbol{A}}_1 & \boldsymbol{E} & \boldsymbol{P}\bar{\boldsymbol{H}} & \bar{\boldsymbol{E}}^{\mathrm{T}} & \bar{\boldsymbol{B}}^{\mathrm{T}} & \bar{\boldsymbol{Q}}^{\mathrm{T}} \\ \bar{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & \bar{\boldsymbol{E}}_3^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{E}^{\mathrm{T}} & \boldsymbol{0} & -\boldsymbol{S}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{\boldsymbol{H}}^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon^{-1}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{\boldsymbol{E}} & \bar{\boldsymbol{E}}_3 & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{\boldsymbol{B}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{P}^{-1} & \boldsymbol{0} \\ \bar{\boldsymbol{Q}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I} \end{bmatrix} < 0。\tag{11}$$

由于不等式(11)含有未知变量 $\boldsymbol{P}$,$\boldsymbol{A}_{\mathrm{c}}$,$\boldsymbol{B}_{\mathrm{c}}$,$\boldsymbol{C}_{\mathrm{c}}$,需应用变量替换法进行转换。将矩阵 $\boldsymbol{P}$ 和 $\boldsymbol{P}^{-1}$ 分块为 $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{Y} & \boldsymbol{N} \\ \boldsymbol{N}^{\mathrm{T}} & \boldsymbol{W} \end{bmatrix}$,$\boldsymbol{P}^{-1} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{M} \\ \boldsymbol{M}^{\mathrm{T}} & \boldsymbol{Z} \end{bmatrix}$,$\boldsymbol{X},\boldsymbol{Y} \in \mathbf{R}^{n\times n}$ 是对称矩阵,$\boldsymbol{W},\boldsymbol{Z}$ 为任意对称矩阵。由 $\boldsymbol{PP}^{-1} = \boldsymbol{I}$ 可得,$\boldsymbol{MN}^{\mathrm{T}} = \boldsymbol{I} - \boldsymbol{XY}$。

定义 $\boldsymbol{F}_1 = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{M}^{\mathrm{T}} & 0 \end{bmatrix}$,$\boldsymbol{F}_2 = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{Y} \\ \boldsymbol{0} & \boldsymbol{N}^{\mathrm{T}} \end{bmatrix}$,由 $\boldsymbol{PF}_1 = \boldsymbol{F}_2$ 可得,$\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{PF}_1 = \boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{F}_2 = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix}$,因此,$\boldsymbol{P} > 0$ 可以保证 $\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} > 0$。对不等式(11) 分别左乘 $\mathrm{diag}\{\boldsymbol{F}_1^{\mathrm{T}},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$,右乘 $\mathrm{diag}\{\boldsymbol{F}_1,\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$,得到:

$$\begin{bmatrix} \boldsymbol{F}_1^{\mathrm{T}}\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{PF}_1 + \boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}\boldsymbol{F}_1 & \boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1 & \boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{E} & \boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{H}} & \boldsymbol{F}_1^{\mathrm{T}}\bar{\boldsymbol{E}}^{\mathrm{T}} & \boldsymbol{F}^{\mathrm{T}}\bar{\boldsymbol{B}}^{\mathrm{T}} & \boldsymbol{F}_1^{\mathrm{T}}\bar{\boldsymbol{Q}}^{\mathrm{T}} \\ \bar{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{PF}_1 & -\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & \bar{\boldsymbol{E}}_3^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}_1 & \boldsymbol{0} & -\boldsymbol{S}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{\boldsymbol{H}}^{\mathrm{T}}\boldsymbol{PF}_1 & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon^{-1}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{\boldsymbol{E}}\boldsymbol{F}_1 & \bar{\boldsymbol{E}}_3 & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{\boldsymbol{B}}\boldsymbol{F}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{P}^{-1} & \boldsymbol{0} \\ \bar{\boldsymbol{Q}}\boldsymbol{F}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I} \end{bmatrix} < 0,\tag{12}$$

其中 $\boldsymbol{E} = [\boldsymbol{I} \quad \boldsymbol{0}]^{\mathrm{T}}$。对式(12)进行变量替换,可得:

$$\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}\boldsymbol{F}_1 = \boldsymbol{F}_2^{\mathrm{T}}\bar{\boldsymbol{A}}\boldsymbol{F}_1 = \begin{bmatrix} \boldsymbol{AX} + \boldsymbol{B}\hat{\boldsymbol{C}} & \boldsymbol{A} \\ \hat{\boldsymbol{A}} & \boldsymbol{YA} + \hat{\boldsymbol{B}}\boldsymbol{C} \end{bmatrix},\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1 = \boldsymbol{F}_2^{\mathrm{T}}\bar{\boldsymbol{A}}_1 = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{YA}_1 + \hat{\boldsymbol{B}}\boldsymbol{C}_1 \end{bmatrix},\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{H}} = \boldsymbol{F}_2^{\mathrm{T}}\bar{\boldsymbol{H}} =$$

$$\begin{bmatrix} \boldsymbol{H}_1 \\ \boldsymbol{YH}_1 + \hat{\boldsymbol{B}}\boldsymbol{H}_2 \end{bmatrix},\boldsymbol{F}_1^{\mathrm{T}}\bar{\boldsymbol{E}}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{XE}_1^{\mathrm{T}} + \hat{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{E}_2^{\mathrm{T}} \\ \boldsymbol{E}_1^{\mathrm{T}} \end{bmatrix},\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{E} = \begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{I} \end{bmatrix},\boldsymbol{F}_1^{\mathrm{T}}\bar{\boldsymbol{B}}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{XG}^{\mathrm{T}} & \boldsymbol{0} \\ \boldsymbol{G}^{\mathrm{T}} & \boldsymbol{0} \end{bmatrix},\boldsymbol{F}_1^{\mathrm{T}}\bar{\boldsymbol{Q}}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{XQ}^{\frac{1}{2}} & \hat{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{R}^{\frac{1}{2}} \\ \boldsymbol{Q}^{\frac{1}{2}} & \boldsymbol{0} \end{bmatrix},$$

其中,$\hat{\boldsymbol{A}}=\boldsymbol{YAX}+\boldsymbol{NB}_c\boldsymbol{CX}+\boldsymbol{YBC}_c\boldsymbol{M}^{\mathrm{T}}+\boldsymbol{NA}_c\boldsymbol{M}^{\mathrm{T}}+\boldsymbol{NB}_c\boldsymbol{DC}_c\boldsymbol{M}^{\mathrm{T}}$,$\hat{\boldsymbol{B}}=\boldsymbol{NB}_c$,$\hat{\boldsymbol{C}}=\boldsymbol{C}_c\boldsymbol{M}^{\mathrm{T}}$,所以式(12)等价于$\boldsymbol{\Phi}<\boldsymbol{0}$,定理得证。

注:利用LMI工具箱求得矩阵$\boldsymbol{X},\boldsymbol{Y},\hat{\boldsymbol{A}},\hat{\boldsymbol{B}},\hat{\boldsymbol{C}},\boldsymbol{M},\boldsymbol{Z}$后,由$\boldsymbol{MN}^{\mathrm{T}}=\boldsymbol{I}-\boldsymbol{XY}$,得到矩阵$\boldsymbol{N}$,进而得到控制器参数分别为:$\boldsymbol{A}_c=\boldsymbol{N}^{-1}(\hat{\boldsymbol{A}}-\boldsymbol{YAX}-\boldsymbol{NB}_c\boldsymbol{CX}-\boldsymbol{YBC}_c\boldsymbol{M}^{\mathrm{T}}-\boldsymbol{NB}_c\boldsymbol{DC}_c\boldsymbol{M}^{\mathrm{T}})\boldsymbol{M}^{-\mathrm{T}}$, $\boldsymbol{B}_c=\boldsymbol{N}^{-1}\hat{\boldsymbol{B}}$, $\boldsymbol{C}_c=\hat{\boldsymbol{C}}\boldsymbol{M}^{-\mathrm{T}}$。

3 仿真算例

考虑随机时滞系统(1),其中,$\boldsymbol{H}_2=0,\boldsymbol{E}_2=0,\boldsymbol{E}_3=0,R=0.1,d=1,t\in[-d,0]$,

$$\boldsymbol{A}=\begin{bmatrix}-5.0 & 5.0\\ -0.1 & -4.0\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}-0.02 & 0.02\\ 0 & 0.05\end{bmatrix},\ \boldsymbol{A}_1=\begin{bmatrix}-0.020 & -0.1\\ 0.004 & 0.5\end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix}0.01 & 0\\ 0.01 & 0\end{bmatrix},$$

$$\boldsymbol{C}_1=\begin{bmatrix}-0.05 & -0.01\\ 0.01 & 0.20\end{bmatrix},\ \boldsymbol{D}=\begin{bmatrix}-0.01 & 0.01\\ 0.01 & -0.10\end{bmatrix},\ \boldsymbol{G}=\begin{bmatrix}-0.01 & 0.01\\ 0.10 & -0.10\end{bmatrix},\ \boldsymbol{H}_1=\begin{bmatrix}0.4 & 0\\ 0 & 0.5\end{bmatrix},$$

$$\boldsymbol{E}_1=\begin{bmatrix}-0.10 & 0.07\\ 0.08 & 0.60\end{bmatrix},\ \boldsymbol{Q}=\begin{bmatrix}0.005 & 0\\ 0 & 0.006\end{bmatrix},\ \boldsymbol{S}=\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}。$$

求解线性矩阵不等式(8)和(9),得到

$$\boldsymbol{X}=\begin{bmatrix}2.3684 & 2.0274\\ 2.0274 & 3.9501\end{bmatrix},\ \boldsymbol{Y}=\begin{bmatrix}25.3547 & -14.1460\\ -14.1460 & 31.8399\end{bmatrix},\ \hat{\boldsymbol{A}}=\begin{bmatrix}0.7875 & 3.4362\\ -3.6128 & -5.5359\end{bmatrix},$$

$$\hat{\boldsymbol{B}}=\begin{bmatrix}-1.72510 & 47.9547\\ -9.22217 & -87.1365\end{bmatrix},\ \hat{\boldsymbol{C}}=\begin{bmatrix}0.0042 & -0.0192\\ -0.0345 & 0.1892\end{bmatrix}。$$

从而,进一步得到控制器参数分别为

$$\boldsymbol{A}_c=\begin{bmatrix}-5.0964 & 5.1530\\ -0.1219 & -3.9890\end{bmatrix},\ \boldsymbol{B}_c=\begin{bmatrix}0.1553 & 0.2627\\ -0.2149 & -1.2240\end{bmatrix},\ \boldsymbol{C}_c=\begin{bmatrix}-0.0212 & 0.0206\\ 0.1983 & -0.1976\end{bmatrix}。$$

假设输出反馈控制器的初始状态为$\hat{\boldsymbol{x}}(0)=[0.01\quad 0.02]^{\mathrm{T}}$,当$\varepsilon=1$时,求得闭环系统的性能指标最小上界为133.118 1,仿真运行结果如图1、2所示。图1显示系统状态在控制器$\boldsymbol{u}(t)$的作用下是依概率全局渐近稳定,图2中的控制器变化曲线经过较短时间波动最终趋于零,这表明本文提出的输出反馈保性能控制器设计方法是有效的。

4 结语

针对具有不确定性、时滞和外部干扰的随机系统,设计输出反馈保性能控制器。通过应用随机系统Lyapunov稳定性理论和Itô公式,在保证系统依概率全局渐近稳定的基础上,使系统的性能指标小于某个上界。此外,利用LMI工具箱求解矩阵不等式,得到最优输出反馈保性能控制器的参数和相应性能指标的最小上界。

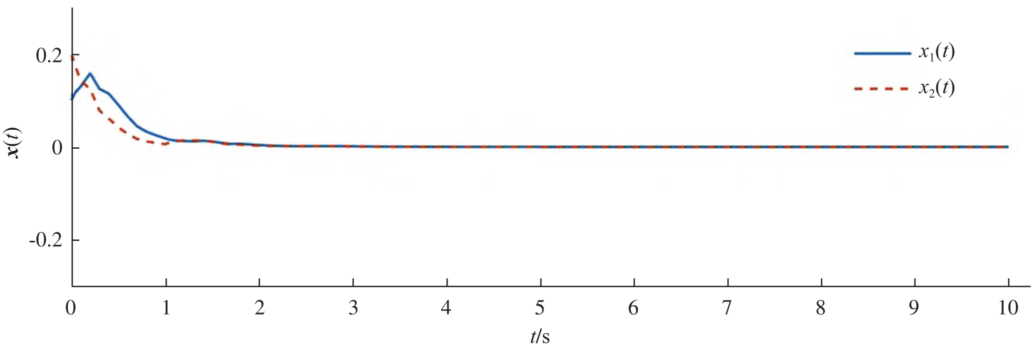

图1 系统状态响应曲线

Fig.1 The response curves of system states

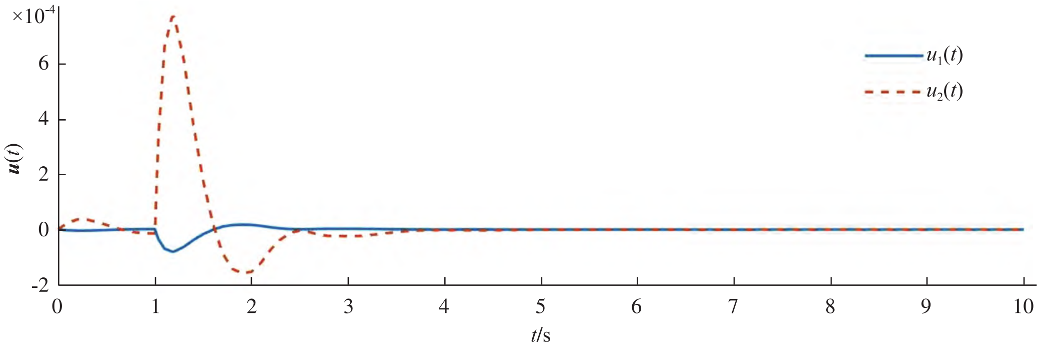

图 2　控制器响应曲线

Fig.2 The response curves of control inputs

参考文献:

[1] CHANG S,PENG T.Adaptive guaranteed cost control of systems with uncertain parameters[J].IEEE Transactions on Automatic Control,1972,17(4):474-483.

[2] PETERSEN I R,MCFARLANE D C.Optimal guaranteed cost control and filtering for uncertain linear systems[J].IEEE Transactions on Automatic Control,1994,39(9):1971-1977.

[3] PETERSEN I R.Guaranteed cost LQG control of uncertain linear systems[J].IEE Proceedings-Control Theory and Applications,1995,142(2):95-102.

[4] MOHEIMANI S O R,PETERSEN L R.Optimal guaranteed cost control of uncertain systems via static and dynamic output feedback[J].Automatica,1996,32(4):575-579.

[5] MA Y C,ZHANG Q L,REN Y W,et al.H_∞ guaranteed cost control for time-delay uncertain discrete systems[C]//International Conference on Control,Automation,Robotics and Vision,2006.

[6] LI H F,ZHOU J.Delay-independent robust guaranteed-cost control for uncertain linear neutral systems[J].Journal of Systems Engineering and Electronics,2007,18(4):858-864.

[7] ZHENG Q X,CHEN H L,XU S Y.Robust guaranteed cost control for uncertain discrete-time systems with state and input quantizations[J].Information Sciences,2021,564:288-305.

[8] LIEN C H.Guaranteed cost observer-based controls for a class of uncertain neutral time-delay systems[J].Journal of Optimization Theory & Applications,2005,126(1):137-156.

[9] 余丽莎,邓飞其.不确定线性时滞随机系统的最优保性能控制[J].自动化技术与应用,2008,27(2):16-19.

[10] CUI P,ZHANG C H,WANG M.Guaranteed cost control of stochastic uncertain systems with time-varying delays[C]// The 6th World Congress on Intelligent Control and Automation,2006.

[11] ZHANG J H,SHI P,QIU J Q.Non-fragile guaranteed cost control for uncertain stochastic nonlinear time-delay systems[J].Journal of the Franklin Institute,2009,346(7):676-690.

[12] ZHU Q X,SARAVANAKUMAR T,GOMATHI S,et al.Finite-time extended dissipative based optimal guaranteed cost resilient control for switched neutral systems with stochastic actuator failures[J].IEEE Access,2019,7:90289-90303.

[13] KARTHICK S A,SAKTHIVEL R,MA Y K,et al.Observer based guaranteed cost control for Markovian jump stochastic neutral-type neural networks[J].Chaos,Solitons & Fractals,2020,133:1-13.

[14] 高振斌,钱富才,刘丁.基于 LMI 的不确定随机系统输出反馈保性能控制[J].信息与控制,2008,37(4):385-390.

[15] 高振斌,钱富才.基于 LMI 的不确定离散随机系统输出反馈保性能控制[J].控制与决策,2011,26(2):170-174.

[16] CHEN W H,GUAN Z H,LU X.Delay-dependent output feedback guaranteed cost control for uncertain time-delay systems[J].Automatica,2004,40(7):1263-1268.

[17] 俞立.鲁棒控制:线性矩阵不等式处理方法[M].北京:清华大学出版社,2002.

[18] 徐胜元,杨成梧.一类不确定性广义非线性系统的鲁棒控制[J].控制理论与应用,2000,17(4):624-626.

[19] OKSENDAL B.Stochastic differential equations: an introduction with applications[M].6nd ed.Heidelberg:Springer-Verlag,2003.

Output Feedback Guaranteed Performance Control for Uncertain Stochastic Time-delay Systems Based on LMI

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Abstract: The guaranteed cost control for a class of norm-bounded stochastic time-delay systems with certain performance indices is studied. The main purpose of this paper is to obtain the optimal output feedback controller which can achieve guaranteed cost control of the system. By combining Lyapunov stability theory of stochastic systems with LMI technology, the guaranteed cost control problem was transformed into a feasible solution problem under LMIs constraints. The controller design scheme proposed in this paper can ensure that the system is globally asymptotically stable according to probability and achieve the requirements of system performance index. In addition, a numerical example was given to prove the feasibility of the proposed design scheme.

Keywords: Linear Matrix Inequality (LMI); guaranteed cost control; stochastic time-delay system; output feedback

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(上接第139页)

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The Spatial and Temporal Distribution Characteristics and Influencing Factors of Homestays in Yantai City

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Abstract: As a new form of tourism, homestays play an important role in the development of rural tourism and rural revitalization. Using the methods of standard deviation, coefficient of variation, buffer analysis and geographic detectors, the spatial distribution of homestays in Yantai from 2011 to 2020 and the change characteristics in the time dimension were analyzed, and the influencing factors were detected. The results are as follows. (1) In terms of time distribution characteristics, the overall number of homestays in Yantai is increasing year by year; the gap in the number of homestays in different regions is increasing, and the level of equilibrium is declining; the development of homestays has shifted from quantitative expansion to quality improvement. (2) In terms of spatial distribution features, on the whole, the development of homestays is extremely unbalanced, forming a spatial pattern of high in the middle and low on both sides, more in the north and less in the south; from the perspective of spatial dynamic changes, it presents a trend of increasing from east to west and decreasing from north to south. (3) The temporal and spatial evolution of homestays in Yantai is greatly affected by factors such as natural environment, regional economic development level, location traffic and tourism resources. On the basis of the above analysis, this paper puts forward suggestions on the high-quality development of homestays in Yantai.

Keywords: homestay; space-time evolution; geographic detector; Yantai City

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