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一类随机时滞系统的有限时间 H_∞ 滤波

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摘要:本文主要研究一类带有界干扰的随机时滞系统的有限时间 H_∞ 滤波问题。将有限时间引入带时滞的 H_∞ 滤波问题中,通过构造合适的李雅普诺夫泛函,结合 Itô 公式得出随机时滞系统具有有限时间 H_∞ 性能的滤波器的设计算法,使得滤波器的误差系统有限时间随机有界。同时,利用 LMI 工具箱求出满足有限时间 H_∞ 性能的滤波器参数。最后数值算例说明本文方法的可行性。

关键词:随机时滞系统;Itô 公式;有限时间随机有界;H_∞ 滤波
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本文考虑如下时滞随机系统:

$$\begin{cases} \mathrm{d}\boldsymbol{x}(t)=((\boldsymbol{A}_0+\Delta\boldsymbol{A})\boldsymbol{x}(t)+\boldsymbol{A}_\mathrm{d}\boldsymbol{x}(t-\tau)+(\boldsymbol{B}_0+\Delta\boldsymbol{B})\boldsymbol{v}(t))\mathrm{d}t+(\boldsymbol{C}_0+\Delta\boldsymbol{C})\boldsymbol{x}(t)\mathrm{d}\omega_0(t), \\ \mathrm{d}\boldsymbol{y}(t)=(\boldsymbol{A}_1\boldsymbol{x}(t)+\boldsymbol{B}_1\boldsymbol{v}(t))\mathrm{d}t+\boldsymbol{C}_1\boldsymbol{x}(t)\mathrm{d}\omega_1(t), \\ z(t)=\boldsymbol{M}\boldsymbol{x}(t), \end{cases} \tag{1}$$

其中:$\boldsymbol{x}(t)\in\mathbf{R}^n$ 为系统的状态,$\boldsymbol{y}(t)\in\mathbf{R}^r$ 为输出,$z(t)\in\mathbf{R}^m$ 为测量输出信号,$\boldsymbol{v}(t)$ 为外界干扰信号,τ 为状态延迟;$\omega_0(t)$,$\omega_1(t)$ 为定义在概率空间(Ω,F,F_t,P)上相互独立的标准布朗运动,$F_t=\sigma\{(\omega_0(s),\omega_1(s)):0\leqslant s\leqslant t\}$;$\boldsymbol{A}_0,\boldsymbol{A}_\mathrm{d},\boldsymbol{B}_0,\boldsymbol{C}_0,\boldsymbol{A}_1,\boldsymbol{B}_1,\boldsymbol{C}_1,\boldsymbol{M}$ 为具有适当维数的常数矩阵;$\Delta\boldsymbol{A},\Delta\boldsymbol{B},\Delta\boldsymbol{C}$ 范数有界,满足:

$$[\Delta\boldsymbol{A}\quad\Delta\boldsymbol{B}\quad\Delta\boldsymbol{C}]=\boldsymbol{G}\boldsymbol{F}(t)[\boldsymbol{H}_0\quad\boldsymbol{H}_1\quad\boldsymbol{H}_2],\boldsymbol{F}^\mathrm{T}(t)\boldsymbol{F}(t)\leqslant\boldsymbol{I},$$

$\boldsymbol{G},\boldsymbol{H}_0,\boldsymbol{H}_1,\boldsymbol{H}_2$ 为适当维数的常数矩阵,矩阵 $\boldsymbol{F}(t)$ 的各个元素是 Lebesgue 可测。

近年来有关随机控制系统的研究逐渐增多,实际工程对系统的控制精度要求更高。由于外部干扰在很大程度上影响系统的状态估计,为获取更准确的系统状态信息,可以采用滤波的方式从量测输出中估计系统真实的状态信息。H_∞ 滤波器[1]是目前常用的滤波器,有关随机控制系统 H_∞ 滤波取得许多研究成果。文献[2]针对多频率区间的 Delta 算子系统,设计了降阶 H_∞ 滤波器;文献[3]研究了一类非线性奇异时滞系统的 H_∞ 滤波问题;文献[4—5]通过构造新的李雅普诺夫泛函,分别对带马尔可夫跳变的中立型混合时滞系统设计了 H_∞ 滤波器;文献[6—7]研究了具有范数有界参数不确定性的中立型混合时滞系统的 H_∞ 滤波问题。

以上关于 H_∞ 滤波问题的研究是基于李雅普诺夫意义下的随机稳定,服从于具有无限稳定时间的动力系统。然而,网络控制、化工产业等实际应用领域,往往关注系统有限时间内的性能,而无限时间上稳态性能的好坏不能代表某一特定时间稳态性能的好坏。因此,针对系统的暂态性能,文献[8—11]提出了有限时间稳定和有限时间有界的定义,文献[12]设计了一类下三角非线性系统的有限时间输出反馈控制器,文献[13—14]分别研究切换系统和网络系统的有限时间控制问题。

本文基于有限时间有界的定义,对一类带有时滞的随机系统设计 H_∞ 滤波器,同时考虑系统在某一特定时间段内的暂态响应和对外部干扰的抑制效果;通过求解 LMI,设计使系统满足有限时间随机有界

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的 H_∞ 滤波器。

1 预备知识

针对时滞随机系统(1),构造如下形式的滤波器:

$$\begin{cases} \mathrm{d}\hat{\boldsymbol{x}}(t) = (\boldsymbol{A}_\mathrm{f}\hat{\boldsymbol{x}}(t) + \boldsymbol{A}_\mathrm{d}\hat{\boldsymbol{x}}(t-\tau))\,\mathrm{d}t + \boldsymbol{B}_\mathrm{f}\mathrm{d}\boldsymbol{y}(t),\ \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0, \\ \hat{z}(t) = \boldsymbol{M}_\mathrm{f}\hat{\boldsymbol{x}}(t), \end{cases} \tag{2}$$

其中,$\hat{x}(t)$ 为滤波器的状态变量,A_f,$\boldsymbol{B}_\mathrm{f}$,$\boldsymbol{M}_\mathrm{f}$ 为待定矩阵。

定义 $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$,$\tilde{z}(t) = z(t) - \hat{z}(t)$,$\tilde{\boldsymbol{x}}(t) = [\boldsymbol{x}^\mathrm{T}(t) \quad \boldsymbol{e}^\mathrm{T}(t)]^\mathrm{T}$,由系统(1)和滤波器(2),得到关于滤波误差的增广系统:

$$\begin{cases} \mathrm{d}\tilde{\boldsymbol{x}}(t) = (\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_\mathrm{d}\tilde{\boldsymbol{x}}(t-\tau) + \tilde{\boldsymbol{B}}\boldsymbol{v}(t))\,\mathrm{d}t + \tilde{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t)\,\mathrm{d}\omega_0(t) + \tilde{\boldsymbol{D}}\tilde{\boldsymbol{x}}(t)\,\mathrm{d}\omega_1(t), \\ \tilde{z}(t) = \tilde{\boldsymbol{M}}\tilde{\boldsymbol{x}}(t), \end{cases} \tag{3}$$

其中:$\tilde{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_0 + \Delta\boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{A}_0 + \Delta\boldsymbol{A} - \boldsymbol{B}_\mathrm{f}\boldsymbol{A}_1 - \boldsymbol{A}_\mathrm{f} & \boldsymbol{A}_\mathrm{f} \end{bmatrix}$,$\tilde{\boldsymbol{A}}_\mathrm{d} = \begin{bmatrix} \boldsymbol{A}_\mathrm{d} \\ \boldsymbol{0} \end{bmatrix}$,$\tilde{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{B}_0 + \Delta\boldsymbol{B} \\ \boldsymbol{B}_0 + \Delta\boldsymbol{B} - \boldsymbol{B}_\mathrm{f}\boldsymbol{B}_1 \end{bmatrix}$,$\tilde{\boldsymbol{C}} = \begin{bmatrix} \boldsymbol{C}_0 + \Delta\boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{C}_0 + \Delta\boldsymbol{C} & \boldsymbol{0} \end{bmatrix}$,

$\tilde{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{B}_\mathrm{f}\boldsymbol{C}_1 & \boldsymbol{0} \end{bmatrix}$,$\tilde{\boldsymbol{M}} = [\boldsymbol{M} - \boldsymbol{M}_\mathrm{f} \quad \boldsymbol{M}_\mathrm{f}]$。

假设 1 给定常数 $T > 0$,外界干扰信号 $\boldsymbol{v}(t)$ 是时变的,且满足 $E\int_0^T \boldsymbol{v}^\mathrm{T}(t)\boldsymbol{v}(t)\,\mathrm{d}t \leqslant d$,$d > 0$。

引理 1[15] (Schur 补引理) 对于任意对称矩阵 $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}$,$\boldsymbol{M} < 0$ 的充要条件为:$\boldsymbol{D} < 0$,$\boldsymbol{A} - \boldsymbol{B}\boldsymbol{D}^{-1}\boldsymbol{C} < 0$ 或 $\boldsymbol{A} < 0$,$\boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B} < 0$。

引理 2[16] 给定矩阵 $\boldsymbol{M}$,$\boldsymbol{N}$,对于任意满足 $\boldsymbol{F}^\mathrm{T}\boldsymbol{F} \leqslant I$ 的矩阵 $\boldsymbol{F}$,有

$$\boldsymbol{MFN} + \boldsymbol{N}^\mathrm{T}\boldsymbol{F}^\mathrm{T}\boldsymbol{M}^\mathrm{T} \leqslant \boldsymbol{MM}^\mathrm{T} + \boldsymbol{N}^\mathrm{T}\boldsymbol{N}。$$

定义 1[17] 考虑系统(3),给定正定矩阵 $\boldsymbol{R}$,有限时间 $T > 0$ 以及常数 $d > 0$,$c_1 < c_2$,如果 $\boldsymbol{v}(t)$ 满足假设 1,且 $\tilde{\boldsymbol{x}}^\mathrm{T}(0)\boldsymbol{R}\tilde{\boldsymbol{x}}(0) \leqslant c_1 \Rightarrow E(\tilde{\boldsymbol{x}}^\mathrm{T}(t)\boldsymbol{R}\tilde{\boldsymbol{x}}(t)) < c_2$,$\forall t \in [0,T]$,那么称系统(3)关于 $(c_1, c_2, T, \boldsymbol{R}, d)$ 有限时间随机有界。

定义 2[18] 如果系统(3)关于 $(c_1, c_2, T, \boldsymbol{R}, d)$ 是有限时间随机有界的,且在零初始条件下有

$$E\int_0^T \tilde{z}^\mathrm{T}(t)\tilde{z}(t)\,\mathrm{d}t < \gamma^2 E\int_0^T \boldsymbol{v}^\mathrm{T}(t)\boldsymbol{v}(t)\,\mathrm{d}t,$$

则称系统(3)关于 $(c_1, c_2, T, \boldsymbol{R}, d)$ 具有给定的有限时间 H_∞ 性能指标 γ,其中 $\gamma > 0$。

2 主要结果

2.1 有限时间 H_∞ 性能分析

定理 1 如果存在标量 $\alpha \geqslant 0$,$\gamma > 0$,正定矩阵 $\boldsymbol{Q}$,满足:

$$\begin{bmatrix} \boldsymbol{\Pi}_{11} - \alpha\tilde{\boldsymbol{Q}} & \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{B}} & \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{A}}_\mathrm{d} & \tilde{\boldsymbol{M}}^\mathrm{T} \\ \tilde{\boldsymbol{B}}^\mathrm{T}\tilde{\boldsymbol{Q}} & -\gamma^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \tilde{\boldsymbol{A}}_\mathrm{d}^\mathrm{T}\tilde{\boldsymbol{Q}} & \boldsymbol{0} & -\tilde{\boldsymbol{Q}} & \boldsymbol{0} \\ \tilde{\boldsymbol{M}} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I} \end{bmatrix} < 0, \tag{4}$$

$$\frac{1}{\lambda_{\min}(\boldsymbol{Q})}\mathrm{e}^{\alpha T}((1+\tau)\lambda_{\max}(\boldsymbol{Q})c_1 + \gamma^2 d) < c_2, \tag{5}$$

则系统(3)关于($c_1,c_2,T,\boldsymbol{R},d$)是有限时间随机有界的,且具有给定的有限时间 H_∞ 性能指标 γ,其中,$\widetilde{\boldsymbol{Q}}=\boldsymbol{R}^{1/2}\boldsymbol{Q}\boldsymbol{R}^{1/2}$,$\boldsymbol{\Pi}_{11}=\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{A}}+\widetilde{\boldsymbol{A}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}+\widetilde{\boldsymbol{C}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{C}}+\widetilde{\boldsymbol{D}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{D}}+\widetilde{\boldsymbol{Q}}$。

证明　设 $\widetilde{\boldsymbol{Q}}=\boldsymbol{R}^{1/2}\boldsymbol{Q}\boldsymbol{R}^{1/2}(\boldsymbol{Q}>0)$,选取 Lyapunov 泛函 $V(\tilde{\boldsymbol{x}}(t))=V_1(\tilde{\boldsymbol{x}}(t))+V_2(\tilde{\boldsymbol{x}}(t))$,其中,$V_1(\tilde{\boldsymbol{x}}(t))=\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t)$,$V_2(\tilde{\boldsymbol{x}}(t))=\int_{t-\tau}^{t}\tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(s)\,\mathrm{d}s$。由 Itô 公式可得

$$\begin{aligned}LV_1(\tilde{\boldsymbol{x}}(t))=&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{Q}}(\widetilde{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t)+\widetilde{\boldsymbol{A}}_{\mathrm{d}}\tilde{\boldsymbol{x}}(t-\tau)+\widetilde{\boldsymbol{B}}\boldsymbol{v}(t))+(\widetilde{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t)+\widetilde{\boldsymbol{A}}_{\mathrm{d}}\tilde{\boldsymbol{x}}(t-\tau)+\widetilde{\boldsymbol{B}}\boldsymbol{v}(t))^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t)+\\&(\widetilde{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}\widetilde{\boldsymbol{Q}}(\widetilde{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t))+(\widetilde{\boldsymbol{D}}\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}\widetilde{\boldsymbol{Q}}(\widetilde{\boldsymbol{D}}\tilde{\boldsymbol{x}}(t))=\\&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{A}}_{\mathrm{d}}\tilde{\boldsymbol{x}}(t-\tau)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{B}}\boldsymbol{v}(t)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau)\widetilde{\boldsymbol{A}}_{\mathrm{d}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t)+\\&\boldsymbol{v}^{\mathrm{T}}(t)\widetilde{\boldsymbol{B}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{C}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{D}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{D}}\tilde{\boldsymbol{x}}(t),\\LV_2(\tilde{\boldsymbol{x}}(t))=&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t)-\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau)\widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{x}}(t-\tau),\end{aligned}$$

因此,可以得到

$$LV(\tilde{\boldsymbol{x}}(t))=[\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\quad \boldsymbol{v}^{\mathrm{T}}(t)\quad \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau)]\begin{bmatrix}\boldsymbol{\Pi}_{11}&\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{B}}&\widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{A}}_{\mathrm{d}}\\\widetilde{\boldsymbol{B}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}&\boldsymbol{0}&\boldsymbol{0}\\\widetilde{\boldsymbol{A}}_{\mathrm{d}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}&\boldsymbol{0}&-\widetilde{\boldsymbol{Q}}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\\boldsymbol{v}(t)\\\tilde{\boldsymbol{x}}(t-\tau)\end{bmatrix}。\tag{6}$$

根据引理 1,由式(4)、(6)可得

$$LV(\tilde{\boldsymbol{x}}(t))<\alpha V(\tilde{\boldsymbol{x}}(t))+\gamma^2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{v}(t)-\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t),\tag{7}$$

显然,$LV(\tilde{\boldsymbol{x}}(t))<\alpha V(\tilde{\boldsymbol{x}}(t))+\gamma^2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{v}(t)$。对不等式从 0 到 t 积分并取数学期望,进一步可得

$$EV(\tilde{\boldsymbol{x}}(t))<V(\tilde{\boldsymbol{x}}(0))+\alpha\int_0^t EV(\tilde{\boldsymbol{x}}(s))\,\mathrm{d}s+\gamma^2E\int_0^t\boldsymbol{v}^{\mathrm{T}}(s)\boldsymbol{v}(s)\,\mathrm{d}s,t\in[0,T],$$

则由 Gronwall 不等式可得

$$EV(\tilde{\boldsymbol{x}}(t))<V(\tilde{\boldsymbol{x}}(0))\mathrm{e}^{\alpha t}+\gamma^2\mathrm{e}^{\alpha t}E\int_0^t\boldsymbol{v}^{\mathrm{T}}(s)\boldsymbol{v}(s)\,\mathrm{d}s,\tag{8}$$

其中

$$\begin{aligned}V(\tilde{\boldsymbol{x}}(0))\mathrm{e}^{\alpha t}\leqslant&\lambda_{\max}(\boldsymbol{Q})\left(\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)\boldsymbol{R}\tilde{\boldsymbol{x}}(0)+\int_{-\tau}^{0}\tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\tilde{\boldsymbol{x}}(s)\,\mathrm{d}s\right)\mathrm{e}^{\alpha t}\leqslant\\&\lambda_{\max}(\boldsymbol{Q})(c_1+\tau\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)\boldsymbol{R}\tilde{\boldsymbol{x}}(0))\mathrm{e}^{\alpha t}\leqslant(1+\tau)\lambda_{\max}(\boldsymbol{Q})c_1\mathrm{e}^{\alpha T},\end{aligned}\tag{9}$$

将式(9)代入式(8),可得

$$EV(\tilde{\boldsymbol{x}}(t))<(1+\tau)\lambda_{\max}(\boldsymbol{Q})c_1\mathrm{e}^{\alpha T}+\gamma^2d\mathrm{e}^{\alpha T}。$$

由于 $EV(\tilde{\boldsymbol{x}}(t))\geqslant E(\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}^{1/2}\boldsymbol{Q}\boldsymbol{R}^{1/2}\tilde{\boldsymbol{x}}(t))\geqslant\lambda_{\min}(Q)E(\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}\tilde{\boldsymbol{x}}(t))$,所以有

$$E(\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}\tilde{\boldsymbol{x}}(t))<\frac{1}{\lambda_{\min}(\boldsymbol{Q})}\mathrm{e}^{\alpha T}((1+\tau)\lambda_{\max}(\boldsymbol{Q})c_1+\gamma^2d)。$$

由式(5)可得 $E(\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}\tilde{\boldsymbol{x}}(t))<c_2$,根据定义 1,系统(3)关于($c_1,c_2,T,\boldsymbol{R},d$)是有限时间随机有界。

下面证明系统(3)具有给定的有限时间 H_∞ 性能指标 γ。对式(7)从 0 到 t 积分并取数学期望,$t\in[0,T]$,进一步有

$$EV(\tilde{\boldsymbol{x}}(t))<\mathrm{e}^{\alpha t}\left(\gamma^2E\int_0^t\boldsymbol{v}^{\mathrm{T}}(s)\boldsymbol{v}(s)\,\mathrm{d}s-E\int_0^t\tilde{\boldsymbol{z}}^{\mathrm{T}}(s)\tilde{\boldsymbol{z}}(s)\,\mathrm{d}s\right)。$$

又由于 $EV(\tilde{\boldsymbol{x}}(t))>0$,所以得到

$$E\int_0^T\tilde{\boldsymbol{z}}^{\mathrm{T}}(s)\tilde{\boldsymbol{z}}(s)\,\mathrm{d}s<\gamma^2E\int_0^T\boldsymbol{v}^{\mathrm{T}}(s)\boldsymbol{v}(s)\,\mathrm{d}s。$$

根据定义 2,系统(3)具有给定的有限时间 H_∞ 性能指标 γ。定理 1 得证。

2.2　有限时间 H_∞ 滤波器设计

定理 2　给定 $\gamma>0$,若存在标量 $\alpha\geqslant0$ 和正定矩阵 $\boldsymbol{Q}_{11}$,$\boldsymbol{Q}_{22}$,以及矩阵 $\boldsymbol{X}$,$\boldsymbol{Y}$,$\boldsymbol{M}_{\mathrm{f}}$,满足下列矩阵不等式:

$$\begin{bmatrix} \boldsymbol{Y}_{11} & \boldsymbol{Y}_{12} & \boldsymbol{Y}_{13} & \boldsymbol{Y}_{14} \\ * & \boldsymbol{Y}_{22} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{Y}_{33} & \boldsymbol{Y}_{34} \\ * & * & * & \boldsymbol{Y}_{44} \end{bmatrix} < 0, \tag{10}$$

$$\boldsymbol{I} < \mathrm{diag}\{\boldsymbol{Q}_{11},\boldsymbol{Q}_{22}\} < \lambda \boldsymbol{I}, \tag{11}$$

$$(1+\tau)\lambda c_1 + \gamma^2 d - c_2 \mathrm{e}^{-\alpha T} < 0, \tag{12}$$

其中:

$$\boldsymbol{Y}_{11} = \begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} \\ * & \boldsymbol{\Psi}_{22} \end{bmatrix}, \boldsymbol{Y}_{12} = \begin{bmatrix} \boldsymbol{\Psi}_{13} & \tilde{\boldsymbol{Q}}_{11}\boldsymbol{A}_{\mathrm{d}} & \boldsymbol{0} & \boldsymbol{\Psi}_{16} \\ \boldsymbol{\Psi}_{23} & \boldsymbol{0} & \tilde{\boldsymbol{Q}}_{22}\boldsymbol{A}_{\mathrm{d}} & \boldsymbol{M}_{\mathrm{f}}^{\mathrm{T}} \end{bmatrix}, \boldsymbol{Y}_{13} = \begin{bmatrix} \boldsymbol{C}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} & \boldsymbol{C}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{Y}_{14} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{C}^{\mathrm{T}} & \tilde{\boldsymbol{Q}}_{11}\boldsymbol{G} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \tilde{\boldsymbol{Q}}_{22}\boldsymbol{G} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{Y}_{22} = \mathrm{diag}\{\boldsymbol{\Psi}_{33}, -\tilde{\boldsymbol{Q}}_{11}, -\tilde{\boldsymbol{Q}}_{22}, -\boldsymbol{I}\}, \boldsymbol{Y}_{33} = \mathrm{diag}\{-\tilde{\boldsymbol{Q}}_{11}, -\tilde{\boldsymbol{Q}}_{22}\}, \boldsymbol{Y}_{34} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \tilde{\boldsymbol{Q}}_{11}\boldsymbol{G} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \tilde{\boldsymbol{Q}}_{22}\boldsymbol{G} \end{bmatrix},$$

$$\boldsymbol{Y}_{44} = \mathrm{diag}\{-\tilde{\boldsymbol{Q}}_{11}, -\tilde{\boldsymbol{Q}}_{22}, -\boldsymbol{I}, -\boldsymbol{I}\}, \tilde{\boldsymbol{Q}}_{11} = \boldsymbol{R}^{1/2}\boldsymbol{Q}_{11}\boldsymbol{R}^{1/2}, \tilde{\boldsymbol{Q}}_{22} = \boldsymbol{R}^{1/2}\boldsymbol{Q}_{22}\boldsymbol{R}^{1/2},$$

以及

$$\boldsymbol{\Psi}_{11} = \tilde{\boldsymbol{Q}}_{11}\boldsymbol{A}_0 + \boldsymbol{A}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} + \tilde{\boldsymbol{Q}}_{11} - \alpha\tilde{\boldsymbol{Q}}_{11} + \boldsymbol{H}_0^{\mathrm{T}}\boldsymbol{H}_0 + \boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2, \boldsymbol{\Psi}_{12} = \boldsymbol{A}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} - \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}} - \boldsymbol{X}^{\mathrm{T}},$$

$$\boldsymbol{\Psi}_{22} = \boldsymbol{X} + \boldsymbol{X}^{\mathrm{T}} + \tilde{\boldsymbol{Q}}_{22} - \alpha\tilde{\boldsymbol{Q}}_{22}, \boldsymbol{\Psi}_{13} = \tilde{\boldsymbol{Q}}_{11}\boldsymbol{B}_0 + \boldsymbol{H}_0^{\mathrm{T}}\boldsymbol{H}_1, \boldsymbol{\Psi}_{23} = \tilde{\boldsymbol{Q}}_{22}\boldsymbol{B}_0 - \boldsymbol{Y}\boldsymbol{B}_1, \boldsymbol{\Psi}_{33} = -\gamma^2\boldsymbol{I} + \boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{H}_1, \boldsymbol{\Psi}_{16} = \boldsymbol{M}^{\mathrm{T}} - \boldsymbol{M}_{\mathrm{f}}^{\mathrm{T}},$$

则系统(3)是有限时间随机有界的,此时滤波器方程为:

$$\begin{cases} \mathrm{d}\hat{\boldsymbol{x}}(t) = \tilde{\boldsymbol{Q}}_{22}^{-1}\boldsymbol{X}\hat{\boldsymbol{x}}(t) + \boldsymbol{A}_{\mathrm{d}}\hat{\boldsymbol{x}}(t-\tau) + \tilde{\boldsymbol{Q}}_{22}^{-1}\boldsymbol{Y}\mathrm{d}\boldsymbol{y}(t), \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0, \\ \hat{z}(t) = \boldsymbol{M}_{\mathrm{f}}\hat{\boldsymbol{x}}(t)。 \end{cases} \tag{13}$$

证明 取 $\tilde{\boldsymbol{Q}} = \mathrm{diag}\{\tilde{\boldsymbol{Q}}_{11},\tilde{\boldsymbol{Q}}_{22}\}$,将各矩阵代入式(4),根据引理1得到

$$\boldsymbol{P} \triangleq \begin{bmatrix} \boldsymbol{\Theta}_{11} & \boldsymbol{\Theta}_{12} \\ * & \boldsymbol{\Theta}_{22} \end{bmatrix} < 0, \tag{14}$$

其中:

$$\boldsymbol{\Theta}_{11} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ * & \boldsymbol{\Sigma}_{22} \end{bmatrix}, \boldsymbol{\Theta}_{12} = \begin{bmatrix} \boldsymbol{\Sigma}_{13} & \tilde{\boldsymbol{Q}}_{11}\boldsymbol{A}_{\mathrm{d}} & \boldsymbol{0} & \boldsymbol{\Sigma}_{16} & \boldsymbol{\Sigma}_{17} & \boldsymbol{\Sigma}_{18} & \boldsymbol{0} & -\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{B}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} \\ \boldsymbol{\Sigma}_{23} & \boldsymbol{0} & \tilde{\boldsymbol{Q}}_{22}\boldsymbol{A}_{\mathrm{d}} & \boldsymbol{M}_{\mathrm{f}}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{\Theta}_{22} = \mathrm{diag}\{-\gamma^2\boldsymbol{I}, -\tilde{\boldsymbol{Q}}_{11}, -\tilde{\boldsymbol{Q}}_{22}, -\boldsymbol{I}, -\tilde{\boldsymbol{Q}}_{11}, -\tilde{\boldsymbol{Q}}_{22}, -\tilde{\boldsymbol{Q}}_{11}, -\tilde{\boldsymbol{Q}}_{22}\},$$

$$\boldsymbol{\Sigma}_{11} = \tilde{\boldsymbol{Q}}_{11}\boldsymbol{A}_0 + \tilde{\boldsymbol{Q}}_{11}\Delta\boldsymbol{A} + \boldsymbol{A}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} + \Delta\boldsymbol{A}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} + \tilde{\boldsymbol{Q}}_{11} - \alpha\tilde{\boldsymbol{Q}}_{11}, \boldsymbol{\Sigma}_{22} = \tilde{\boldsymbol{Q}}_{22}\boldsymbol{A}_{\mathrm{f}} + \boldsymbol{A}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} + \tilde{\boldsymbol{Q}}_{22} - \alpha\tilde{\boldsymbol{Q}}_{22},$$

$$\boldsymbol{\Sigma}_{12} = \boldsymbol{A}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} + \Delta\boldsymbol{A}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} - \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{B}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} - \boldsymbol{A}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22}, \boldsymbol{\Sigma}_{23} = \tilde{\boldsymbol{Q}}_{22}\boldsymbol{B}_0 + \tilde{\boldsymbol{Q}}_{22}\Delta\boldsymbol{B} - \tilde{\boldsymbol{Q}}_{22}\boldsymbol{B}_{\mathrm{f}}\boldsymbol{B}_1, \boldsymbol{\Sigma}_{13} = \tilde{\boldsymbol{Q}}_{11}\boldsymbol{B}_0 + \tilde{\boldsymbol{Q}}_{11}\Delta\boldsymbol{B},$$

$$\boldsymbol{\Sigma}_{16} = \boldsymbol{M}^{\mathrm{T}} - \boldsymbol{M}_{\mathrm{f}}^{\mathrm{T}}, \boldsymbol{\Sigma}_{17} = \boldsymbol{C}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} + \Delta\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11}, \boldsymbol{\Sigma}_{18} = \boldsymbol{C}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} + \Delta\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22}。$$

将 $\boldsymbol{P}$ 分解为确定性和不确定性两部分,即 $\boldsymbol{P} = \boldsymbol{Z} + \boldsymbol{Z}_1$,其中:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} \\ * & \boldsymbol{\Theta}_{22} \end{bmatrix}, \boldsymbol{Z}_1 = \begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ * & \boldsymbol{0} \end{bmatrix}, \boldsymbol{\Gamma}_{11} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} \\ * & \boldsymbol{\Sigma}_{22} \end{bmatrix},$$

$$\boldsymbol{\Gamma}_{12} = \begin{bmatrix} \tilde{\boldsymbol{Q}}_{11}\boldsymbol{B}_0 & \tilde{\boldsymbol{Q}}_{11}\boldsymbol{A}_{\mathrm{d}} & \boldsymbol{0} & \boldsymbol{\Sigma}_{16} & \boldsymbol{C}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} & \boldsymbol{C}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} & \boldsymbol{0} & -\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{B}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} \\ \boldsymbol{\Xi}_{23} & \boldsymbol{0} & \tilde{\boldsymbol{Q}}_{22}\boldsymbol{A}_{\mathrm{d}} & \boldsymbol{M}_{\mathrm{f}}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{\Lambda}_{11} = \begin{bmatrix} \tilde{\boldsymbol{Q}}_{11}\Delta\boldsymbol{A} + \Delta\boldsymbol{A}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} & \Delta\boldsymbol{A}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} \\ \tilde{\boldsymbol{Q}}_{22}\Delta\boldsymbol{A} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{\Lambda}_{12} = \begin{bmatrix} \tilde{\boldsymbol{Q}}_{11}\Delta\boldsymbol{B} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Delta\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} & \Delta\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} & \boldsymbol{0} & \boldsymbol{0} \\ \tilde{\boldsymbol{Q}}_{22}\Delta\boldsymbol{B} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{\Xi}_{11} = \tilde{\boldsymbol{Q}}_{11}\boldsymbol{A}_0 + \boldsymbol{A}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} + \tilde{\boldsymbol{Q}}_{11} - \alpha\tilde{\boldsymbol{Q}}_{11}, \boldsymbol{\Xi}_{23} = \tilde{\boldsymbol{Q}}_{22}\boldsymbol{B}_0 - \tilde{\boldsymbol{Q}}_{22}\boldsymbol{B}_{\mathrm{f}}\boldsymbol{B}_1, \boldsymbol{\Xi}_{12} = \boldsymbol{A}_0^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} - \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{B}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} - \boldsymbol{A}_{\mathrm{f}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22}。$$

令 $\boldsymbol{Z}_1 = \boldsymbol{T}_1\boldsymbol{F}\boldsymbol{R}_1 + \boldsymbol{R}_1^{\mathrm{T}}\boldsymbol{F}\boldsymbol{T}_1^{\mathrm{T}} + \boldsymbol{T}_2\boldsymbol{F}\boldsymbol{R}_2 + \boldsymbol{R}_2^{\mathrm{T}}\boldsymbol{F}\boldsymbol{T}_2^{\mathrm{T}}$,其中 $\boldsymbol{T}_1 = [\boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} \quad \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} \quad \boldsymbol{0}]^{\mathrm{T}}$,$\boldsymbol{R}_2 = [\boldsymbol{H}_2 \quad \boldsymbol{0}]$,$\boldsymbol{R}_1 = [\boldsymbol{H}_0 \quad \boldsymbol{0} \quad \boldsymbol{H}_1 \quad \boldsymbol{0}]$,$\boldsymbol{T}_2 = [\boldsymbol{0} \quad \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{11} \quad \boldsymbol{G}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_{22} \quad \boldsymbol{0} \quad \boldsymbol{0}]^{\mathrm{T}}$,则

$$P = Z + T_1FR_1 + R_1^{\mathrm T}FT_1^{\mathrm T} + T_2FR_2 + R_2^{\mathrm T}FT_2^{\mathrm T} < 0。$$

由引理 2 可得

$$Z + T_1FR_1 + R_1^{\mathrm T}FT_1^{\mathrm T} + T_2FR_2 + R_2^{\mathrm T}FT_2^{\mathrm T} < Z + T_1T_1^{\mathrm T} + R_1^{\mathrm T}R_1 + T_2T_2^{\mathrm T} + R_2^{\mathrm T}R_2 < 0。$$

将 T_1, R_1, T_2, R_2 代入,整理得

$$Z + T_1T_1^{\mathrm T} + R_1^{\mathrm T}R_1 + T_2T_2^{\mathrm T} + R_2^{\mathrm T}R_2 = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix}, \tag{15}$$

其中:

$$\Omega_{11} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix}, \Omega_{22} = \mathrm{diag}\{\Phi_{33}, -\tilde Q_{11}, -\tilde Q_{22}, -I, \Phi_{77}, \Phi_{88}, -\tilde Q_{11}, -\tilde Q_{22}\},$$

$$\Omega_{12} = \begin{bmatrix} \Phi_{13} & \tilde Q_{11}A_{\mathrm d} & 0 & \Sigma_{16} & C_0^{\mathrm T}\tilde Q_{11} & C_0^{\mathrm T}\tilde Q_{22} & 0 & -C_1^{\mathrm T}B_{\mathrm f}^{\mathrm T}\tilde Q_{22} \\ \Xi_{23} & 0 & \tilde Q_{22}A_{\mathrm d} & M_{\mathrm f}^{\mathrm T} & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{11} = \Xi_{11} + \tilde Q_{11}GG^{\mathrm T}\tilde Q_{11} + H_0^{\mathrm T}H_0 + H_2^{\mathrm T}H_2, \Phi_{12} = \Xi_{12} + \tilde Q_{11}GG^{\mathrm T}\tilde Q_{22}, \Phi_{22} = \Sigma_{22} + \tilde Q_{22}GG^{\mathrm T}\tilde Q_{22},$$

$$\Phi_{13} = \tilde Q_{11}B_0 + H_0^{\mathrm T}H_1, \Phi_{33} = -\gamma^2 I + H_1^{\mathrm T}H_1, \Phi_{77} = -\tilde Q_{11} + \tilde Q_{11}GG^{\mathrm T}\tilde Q_{11}, \Phi_{88} = -\tilde Q_{22} + \tilde Q_{22}GG^{\mathrm T}\tilde Q_{22}。$$

令 $X = \tilde Q_{22}A_{\mathrm f}, Y = \tilde Q_{22}B_{\mathrm f}$,由引理 1,可以将不等式(15)转化为不等式(10),由式(11)、(12)可得不等式(5)成立,根据定理 1,系统(3)是有限时间随机有界的。

当确定 $\gamma, R, T, c_1, c_2, d, \alpha$ 后,不等式(10)可转化为 LMI,通过求解 LMI 得到滤波器系数。

3　仿真实例

考虑随机时滞系统(1),其中各系数矩阵为:

$$A_0 = \begin{bmatrix} -2.00 & 0.7 \\ 0.11 & -4.5 \end{bmatrix}, A_{\mathrm d} = \begin{bmatrix} -0.2 & 0.3 \\ -0.2 & -0.3 \end{bmatrix}, B_0 = \begin{bmatrix} -0.5 \\ -0.3 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} -0.4 & 0.1 \\ -0.5 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -3 \end{bmatrix}, B_1 = \begin{bmatrix} 0.40 \\ -0.25 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, M = [-1 \quad 1],$$

$$G = [-0.05 \quad 0.15], H_0 = [-0.5 \quad -0.2], H_1 = -0.03, H_2 = [0.01 \quad -0.03], F = I。$$

给定 $\gamma = 1, c_1 = 1, c_2 = 18, d = 1, T = 1, \alpha = 2, \tau = 0.1, R = I$,求解定理 2 中的不等式(10)~(12),可得滤波器的参数:

$$A_{\mathrm f} = \begin{bmatrix} -5.8857 & 1.0000 \\ 1.0000 & -12.1446 \end{bmatrix}, B_{\mathrm f} = \begin{bmatrix} -0.8840 & 0.0174 \\ 0.0174 & -0.2565 \end{bmatrix}, M_{\mathrm f} = [-0.9663 \quad 0.8586]。$$

选取外部干扰信号 $v(t) = -3\sin(t-1)$,系统(1)、(2)初始状态均为 $[0.7 \quad -0.6]^{\mathrm T}$,仿真结果见图 1、2。由图 1 可以看出,当 $0 \leqslant t \leqslant 1$ 时,滤波误差系统状态 $E\left(\tilde x^{\mathrm T}(t)R\tilde x(t)\right) < 18$;图 2 表明,当 $\tilde x^{\mathrm T}(0)R\tilde x(0) \leqslant 1$,始终有 $E\int_0^T \tilde z^{\mathrm T}(t)\tilde z(t)\,\mathrm dt - \gamma^2 E\int_0^T v^{\mathrm T}(t)v(t)\,\mathrm dt < 0$。因此,仿真结果表明系统(3)在给定时间 $[0,1]$ 内随机有界,且具有 H_∞ 性能指标 γ。

4　结语

本文研究一类带有时滞的随机系统的有限时间 H_∞ 滤波问题。根据有限时间随机有界的定义,同时考虑系统的瞬态性能和在某段时间内对外部干扰的抑制能力,通过构造李雅普诺夫泛函,以及利用矩阵变换方法,给出了使系统满足有限时间 H_∞ 滤波器的充分条件,并通过选定合适的常数将其转化为线性矩阵不等式问题。

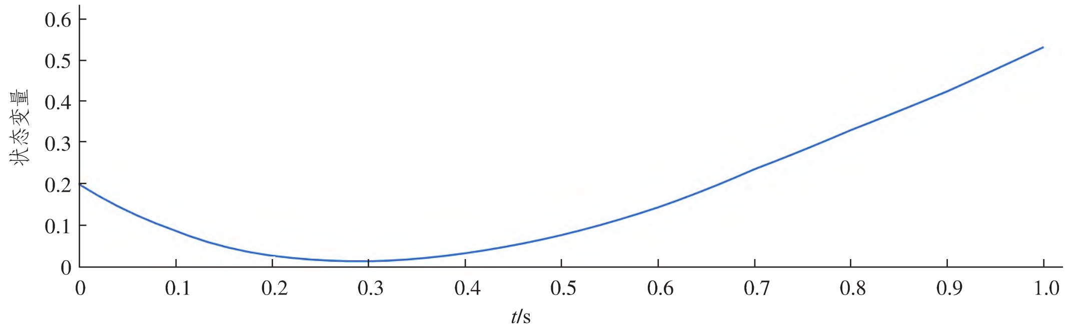

图1　状态变量 $E\left(\tilde{x}^{T}(t)R\tilde{x}(t)\right)$ 变化曲线

Fig. 1 The curve of state variable $E\left(\tilde{x}^{T}(t)R\tilde{x}(t)\right)$

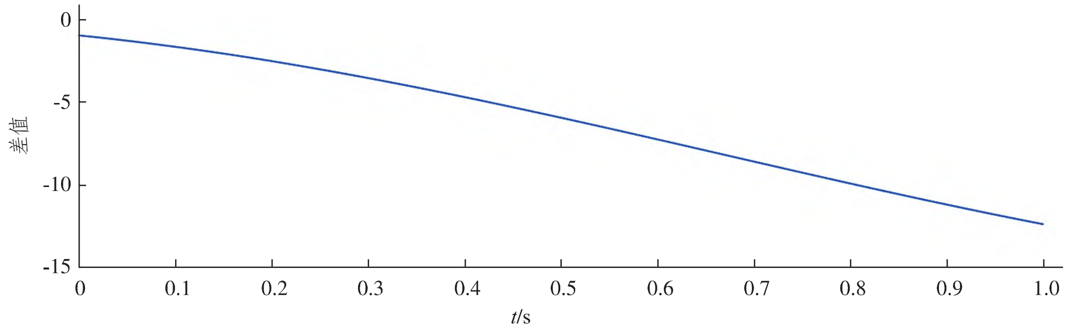

图2　$\tilde{z}^{T}(t)\tilde{z}(t)-v^{T}(t)v(t)$ 变化曲线

Fig. 2 The curve of $\tilde{z}^{T}(t)\tilde{z}(t)-v^{T}(t)v(t)$

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Finite-time H_∞ Filtering for a Class of Stochastic Time-delay Systems

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Abstract: In this paper，the finite-time H_∞ filtering problem is studied for a class of stochastic time-delay systems with bounded disturbances. Integrating finite time problems into H_∞ filtering problems with time-delay，the design algorithm of the filter with finite time H_∞ performance for the stochastic system with time-delay was derived by constructing a suitable Lyapunov functional and combining it with Itô formula，so that the error system of the filter is stochastically bounded in finite time. Furthermore，the filter parameters satisfying the finite-time H_∞ performance were obtained by using LMI toolbox. Finally，the feasibility of the proposed method was illustrated by a numerical example.

Keywords: stochastic time-delay system; Itô formula; finite-time stochastic boundedness; H_∞ filtering

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